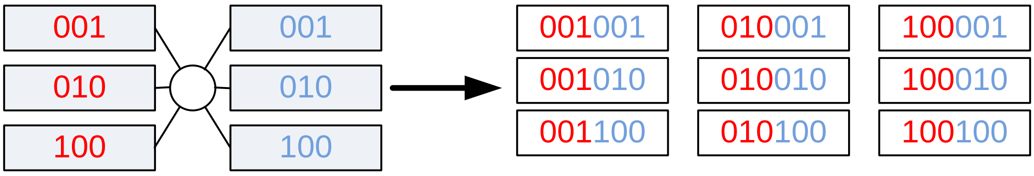
April 16

E-OLYMP [128. Lucky tickets]() Find the number of lucky tickets with the sum of the first three digits *n*. Ticket is called lucky if it is a six-digit number and the sum of the first three digits equals to the sum of the last three digits.

► Compute the amount *p* of three-digit numbers (that can start from zero), which sum of digits is *n*. If you append any three-digit number (which sum of digits is *n*) next to any other three-digit number (which sum of digits is *n*), you get a lucky ticket. The number of lucky tickets is p^2.

Consider the case when $n = 1$. Three-digit numbers which sum of digits is 1, are 001, 010 and 100. If you combine any of them with any other, you'll get 9 lucky tickets.



E-OLYMP [4101. Three digit numbers]() Find all three digit numbers with sum of digits equals to *n*.

► Iterate over the digits of hundreds, tens and ones and find all three-digit numbers, which sum of digits is *n*.

E-OLYMP [7405. Ice cream]() Stepan and his friends went on holiday to Uzhlyandiya. While hiding from the heat, they decided to buy an ice cream. There exist *n* flavors of ice cream, numbered from 1 to *n*. Some tastes are incompatible, such couples should be avoided, otherwise it will be very unpleasant taste. Stepan wants to know the number of ways to choose three different flavors of ice cream so that among them there was no incompatible pair. The order of flavors in triples does not matter.

► Store in the two-dimensional array *mas* the pairs of incompatible tastes: assign mas[*i*][*j*] = 1 if tastes *i* and *j* are incompatible and mas[*i*][*j*] = 0 otherwise.

Find the number of triplets of different tastes in the problem. Using the triple loop iterate through the triples of tastes (*i*, *j*, *k*), where $i < j < k$. For each pair from the triple (*i*, *j*), (*i*, *k*), (*j*, *k*), check whether it is compatible.

For the given example the possible triplets are (1 4 5), (2 3 5) and (2 4 5). The compatibility matrix of tastes has the form:

	1	2	3	4	5
1	0	1	1	0	0
2	1	0	0	0	0
3	1	0	0	1	0
4	0	0	1	0	0
5	0	0	0	0	0

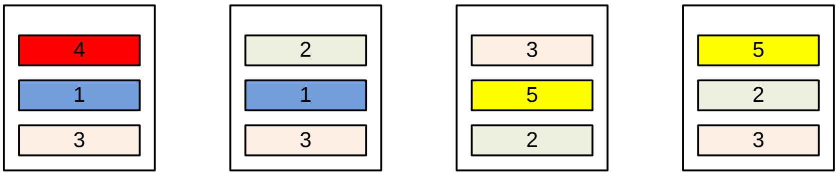
E-OLYMP [7503. Olympiad] n teams arrived to Programming Competition, each team consists of A_i ($1 \le i \le n$) participants. For competition the classes were prepared with the same number m of computers in each one. Find the minimum number of classes required for competition so that each class will contain participants from different teams only. That is, in any class cannot be more than one participant from one team.

► In total, $s = \sum_{i=1}^{n} A_i$ participants arrived at the competition. Each room contains m computers, then you need at least $\lceil s/m \rceil$ rooms. Let p be the largest number of participants from one team (the maximum value among all A_i). If $p > \lceil s/m \rceil$, then we need at least p rooms. It can be shown that max($\lceil s/m \rceil$, p) classes is always enough for the Olympiad.

There are 2 + 3 + 4 + 1 + 2 = 12 schoolchildren arrived at the Olympiad. Each class has m = 3 computers, so for the Olympiad it is necessary to use at least 12 / 3 = 4 classes.

The maximum number of participants is from the third team, it is 4, that can be seated one by one in 4 classes.

For example, the following arrangement of children is possible.



E-OLYMP [9637. Dino and snow buildings] Dino travels to China (the Coronavirus was not yet spread there) and sees 10^9 buildings there. The height of the first building is 1, the second building has the height 1 + 2, the third one is 1 + 2 + 3, etc. The height of the last building is 1 + 2 + 3 + ... + 10^9 (all heights are given in meters).

After a day, Dino fell asleep and saw an interesting view in the morning: it snowed at night and the height of all buildings increased equally! Dino wants to know how many meters of snow fell. To do this, he went to two neighbouring buildings (for example, 3 - d and 4 - th) and measured their new heights. Determine how many meters of snow fell if the heights of these buildings equal to a and b meters respectively.

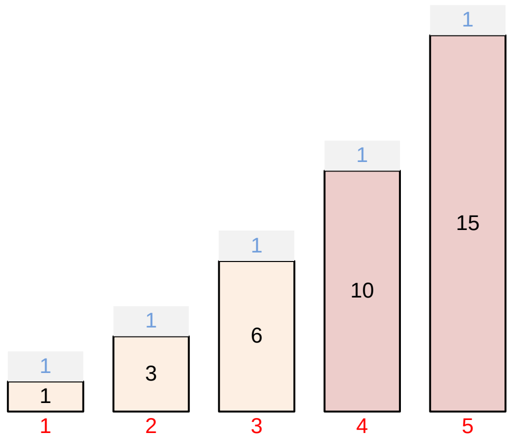
► Let Dino measure the height of snow near buildings with heights $1 + \ldots + i$ and $1 + \ldots + i + (i + 1)$. Let x be the height of the snowfall. Then

$$a = 1 + \ldots + i + x = (1 + i) * i / 2 + x = (i + i^2) / 2 + x,$$

$$b = 1 + \ldots + i + (i + 1) + x = (2 + i) * (i + 1) / 2 + x = (2 + 3i + i^2) / 2 + x$$

We have: $b - a = (2i + 2) / 2 = i + 1$, whence $i = b - a - 1$. The height of the snowfall x is $a - (i + i^2) / 2$.

In the sample $a = 11$, $b = 16$. Then $i = 16 - 11 - 1 = 4$. The height of the snowfall is $11 - (4 + 4^2) / 2 = 11 - 10 = 1$.



E-OLYMP [**123. Number of zeroes in factorial**]() Find the number of zeros at the end of n! (n factorial).

► The factorial of the integer n is equal to the product of integers from 1 to n. Zero at the end of the product appears as a result of multiplication 2 and 5. But since in the prime factorization of n! there are more twos than fives, then the number of zeros at the end of n! equals to the number of fives in factorization of n!. This number is

$$\left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{5^2} \right\rfloor + \left\lfloor \frac{n}{5^3} \right\rfloor + \ldots$$

The summation takes place until the next term equals to 0.

Find the number of zeros ending in 100!

$$\left\lfloor \frac{100}{5} \right\rfloor + \left\lfloor \frac{100}{5^2} \right\rfloor + \left\lfloor \frac{100}{5^3} \right\rfloor + \ldots = 20 + 4 = 24$$

The third term is already equal to zero, since $100 < 5^3 = 125$.

E-OLYMP [**925. Perimeter and area of tiangle**]() The real numbers x_1, y_1, x_2, y_2, x_3, y_3 – the coordinates of triangle vertices are given. Find the perimeter and the area of a triangle.

► Given the coordinates of the vertices of the triangle, calculate the lengths of its sides. Next, calculate the perimeter as the sum of the sides lengths and the area according to Heron’s formula.

E-OLYMP [**932. The height of triangle**]() The area of triangle is S. The length of its base is a greater than its height. Find the height of triangle.

► Let h be the height of triangle. Then its base is $h + a$. The area of the triangle is $S = ½\, h\, (h + a)$. The values of S and a are given, solve the quadratic equation for h:

$$h^2 + ha - 2S = 0,$$

$$\text{discriminant } D = a^2 + 4S,$$

$$\text{the positive root is } h = \frac{-a + \sqrt{D}}{2}$$